B6: Steering Kinematics Derivations (Team). Due Tuesday Jan. 26^{th}.

Derivation time: In this assignment derive the following geometric relationships for the front-end of the bicycle (using and including a picture will be helpful).

- Begin by drawing and labeling the four triangles represented in Figure 1.8 on page 10 of your text.
- Then derive the relations

$$\tan\alpha = \tan\varepsilon\cos\delta, \quad \tan\Delta = \tan\delta\cos\varepsilon, \quad \sin\beta = \sin\alpha\sin\beta.$$

- And then quantities

$$\sin\alpha = \frac{\cos\delta\sin\varepsilon}{\sqrt{\cos^2\delta\sin^2\varepsilon + \cos^2\varepsilon}} \qquad \cos\alpha = \frac{\cos\varepsilon}{\sqrt{\cos^2\delta\sin^2\varepsilon + \cos^2\varepsilon}}$$

or

$$\sin\alpha = \frac{\cos\delta\sin\varepsilon}{\sqrt{1-\sin^2\delta\sin^2\varepsilon}} \qquad \cos\alpha = \frac{\cos\varepsilon}{\sqrt{1-\sin^2\delta\sin^2\varepsilon}}.$$

- Lastly show that the steering head height decreases by

$$\Delta h = R_f\left(1 - \frac{\cos\varepsilon}{\cos\alpha}\right) = R_f\left(1 - \sqrt{1-\sin^2\delta\sin^2\varepsilon}\right) = R_f\left(1 - \sqrt{\cos^2\delta\sin^2\varepsilon + \cos^2\varepsilon}\right)$$

when the handlebars are turned by an angle δ and the fork-offset, d, is 0.

Extra Credit (worth five team +'s): Show that when $d \neq 0$,

$$\begin{aligned}\Delta h = R_f\left(1 - \frac{\cos\varepsilon}{\cos\alpha}\right) - d\sin\varepsilon(1-\cos\delta) &= R_f\left(1 - \sqrt{1-\sin^2\delta\sin^2\varepsilon}\right) - d\sin\varepsilon(1-\cos\delta) \\ &= R_f\left(1 - \sqrt{\cos^2\delta\sin^2\varepsilon + \cos^2\varepsilon}\right) - d\sin\varepsilon(1-\cos\delta).\end{aligned}$$